Operation Approaches on α-(γ,β)-Open(closed) Mappings and γ generalized α-open sets

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Abstract

In this paper the concept of α-(γ,β)-open(closed) mappings have been introduced and studied . Further γ-g α-open (closed) sets, γ-αT_b space, γ-αT_d space and γ-$T_{g\alpha}$ space have been introduced and some of their basic properties are studied.

Key words :(α-γ,β) -continuous mappings, α-(γ,β)- continuous mappings ,γ-$g\alpha$-(open)closed sets, γ-αT_b space,γ-αT_d space, γ-$T_{g\alpha}$ space.

1 Introduction

O.Njastad [6] introduced α-open sets in a topological space and studied some of its properties. Kasahara [3] defined the concept of an operation on topological spaces and introduced α -closed graphs of an operation. Ogata [7] called the operation α as γ operation and introduced the notion of τ_γ which is the collection of all γ-open sets in a topological space (X ,τ). Further he introduced the concept of γ- T$_i$ spaces ($i = 0, \frac{1}{2}, 1, 2$) and characterized γ - T$_i$ spaces using the notion of γ- closed set or γ-open sets. G.Sai sundara Krishnan and N.Kalaivani [9] introduced α-γ-open sets in topological spaces and studied some of their basic properties..

In his paper paper in section 3 we studied some properties of α-(γ,β)-continuous mappings, α-(γ,β)- homeomorphism and we introduced the notion of α-β-T$_i$($i = \frac{1}{2}, 1, 2$) spaces .

In sections 4 and 5 we introduced the concept of α-(γ,β)-open and α-(γ,β)-closed mappings and characterized the mappings with α-γ-interior and α-γ-closure operators and investigated

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their basic properties.

In section 6 γ-$g\alpha$-open sets, γ-$g\alpha$- closed sets ,γ-α T_b, γ-α T_d and γ-$T_{g\alpha}$ space have been introduced and some of their properties are discussed.

2 Preliminaries

In this section we recall some of the basic Definitions and Remarks.

Definition 2.1 [6] Let(X,τ) be a topological space and A be a subset of X. Then A is said to be α- open set if A $\subseteq$ int(cl(int(A)))and α-closed set if cl(int(cl(A))) $\supseteq$ A.

Definition 2.2 [9] Let (X,τ) be a topological space and γ be an operation on τ. Then a subset A of X is said to be a α-γ-open set if and only if $A \subseteq \tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(A)))$

Definition 2.3 [9] Let (X,τ) be a topological space and γ be an operation on τ. Then a subset A of X is said to be a α-γ-closed if and only if X $-$ A is α-γ-open.

Remark 2.4 [9] Let (X,τ) be a topological space and γ be an operation on τ and A be a subset of X . Then A is α-γ- closed if and only if $A \supseteq \tau_\gamma - cl(\tau_\gamma - int(\tau_\gamma - cl(A)))$

Definition 2.5 [9] Let(X ,τ) be a topological space and γ be an operation on τ and A be a subset of X . Then $\tau_{\alpha-\gamma}$- interior of A is the union of all α-γ-open sets contained in A and it is denoted by $\tau_{\alpha-\gamma} - int(A)$. $\tau_{\alpha-\gamma}$-int (A) = $\cup\{U : U \ is \ a \ \alpha - \gamma - open \ set \ and \ U \ \subseteq A\}$

Definition 2.6 [9] Let (X ,τ) be a topological space and γ be an operation on τ. Let A be a subset of X . Then $\tau_{\alpha-\gamma}$-closure of A is the intersection of α-γ- closed sets containing A and it is denoted by $\tau_{\alpha-\gamma}$- cl(A). That is $\tau_{\alpha-\gamma}$- cl(A) = $\cap\{F : F \ is \ a \ \alpha - \gamma - closed \ set \ and \ A \subseteq F\}$

Remark 2.7 [9] Let (X,τ) be a topological space and γ be an operation on τ. A subset A of X is said to be α-γ-generalized closed (written as α-γ g-closed set) if $\tau_{\alpha-\gamma} - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α-γ- open set in (X,τ).

Definition 2.8 [7] A mapping $f : X \to Y$ is said to be (γ,β)- continuous if for each x of X and each open set V containing f(x) there exists an open set U such that x $\in$ U and $f(U^\gamma) \subseteq V^\beta$.

Definition 2.9[7] Let(X,τ) be a topological space and γ be an operation on τ. A subset A of X is said to be γ-generalized closed (written as γ-g.closed set) if cl_γ $\subseteq$ U whenever $A \subseteq U$ and U is γ-open in (X,τ).

Definition 2.10[2] A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be α-(γ,β)- continuous if and only if for any α-β-open set U of Y, $f^{-1}(U)$ is α-γ-open in X.

3 Some properties of α-(γ,β)-continuous mapping and α-β-T_i spaces

Let (X, τ) and (Y, σ) be two topological spaces. $\gamma : \tau \to P(X)$ and $\beta : \sigma \to P(Y)$ be operations on τ and σ respectively.

Definition 3.1 Let (X,τ) be a topological space and $\gamma : \tau \to P(X)$ be an operation on τ. Then a subset A of X is said to be a α-γ- neighbouhood of a point x $\in$ X if there exists a α-γ-open set U such that x $\in$ U $\subseteq$ A.

Theorem 3.2 A mapping f : (X,τ) $\to$ (Y,σ) is α-(γ,β)- continuous if and only if for each x in X, the inverse of every α-β-neighbourhood of f(x) is α-γ-neighbourhood of x.

Proof: Let x $\in$ X and B be a α-β-neighbourhood of f(x). By Definition 3.1 there exists a V $\in \sigma_{\alpha-\beta}$(Y) such that $f(x) \in V \subseteq B$. This implies that $x \in f^{-1}(V) \subseteq f^{-1}(B)$. Since f is α-(γ,β)- continuous, $f^{-1}(V) \in \tau_{\alpha-\gamma}(X)$. Hence $f^{-1}(B)$ is a α-γ-neighbourhood of x.

Conversely, Let $B \in \sigma_{\alpha-\beta}$. Put $A = f^{-1}(B)$. Let x $\in$ A . Then f(x) $\in$ B. B is a α-β-neighbourhood of f(x). So by hypothesis, $A = f^{-1}(B)$ is a α-γ-neighbourhood of x. Hence by Definition 3.1 there exists $A_x \in \tau_{\alpha-\gamma}$ such that x $\in A_x \subseteq$ A. This implies that A = $\cup_{x\in A} A_x$. By Theorem 3.4 [9] A is α-γ-open in X. Therefore f is α-(γ,β)- continuous.

Theorem 3.3 A mapping $f : (X, \tau) \to (Y, \sigma)$ is α-(γ,β)- continuous if and only if for each point x in X and each α-β-neighbourhood B of f(x), there is a α-γ-neighbourhood A of x such that $f(A) \subseteq B$.

Proof: Let x in X and B be a α-β-neighbourhood of f(x). Then there exists $O_{f(x)} \in \sigma_{\alpha-\beta}$ such that f(x)$\in O_{f(x)} \subseteq B$. It follows that x $\in f^{-1}(O_{f(x)}) \subseteq f^{-1}(B)$. By hypothesis, $f^{-1}(O_{f(x)}) \subseteq \tau_{\alpha-\gamma}$. Let $A = f^{-1}(B)$. Then it follows that A is α-γ-neighbourhood of x and $f(A) = f(f^{-1}(B)) \subseteq B$.

Conversely, let U $\in \sigma_{\alpha-\beta}$. Take $W = f^{-1}(U)$. Let x $\in$ W. Then f(x) $\in$ U. Thus U is a α-β-neighbourhood of f(x). By hypothesis, there exists a α-γ- neighbourhood V_x of x such that $f(V_x) \subseteq U$. Thus it follows that x $\in V_x \subseteq f^{-1}(f(V_x)) \subseteq f^{-1}(U) = W$. Since V_x is a α-γ-neighbourhood of x, which implies that there exists a $W_x \in \tau_{\alpha-\gamma}$ such that $x \in W_x \subseteq W$. This implies that W = $\cup_{x\in W} W_x$. By Theorem 3.4 [9] , W is α-γ- open in X. Thus f is α-(γ,β)- continuous.

Theorem 3.4 Let $f : (X, \tau) \to (Y, \sigma)$ be a mapping . Then the following statements are equivalent:
(i) f is α-(γ,β)- continuous.
(ii) $f[\tau_{\alpha-\gamma} - cl(A)] \subseteq \sigma_{\alpha-\beta} - cl[f(A)]$ holds for every subset A of (X,τ).
(iii) For every α-β- closed set V of (Y,σ),$f^{-1}(V)$ is α-γ-closed in (X,τ).

Proof:
$(i) \rightarrow (ii)$. Let y $\in f(\tau_{\alpha-\gamma} - cl(A))$ and V be any α-β-open set containing y. Using Theorem 3.3,then there exists a point x $\in$ X and a α-γ-open set U such that x $\in$ U with f(x) = y and f(U) $\subseteq$ V. Since x $\in \tau_{\alpha-\gamma} - cl(A))$, we have U $\cap A \neq \phi$ and hence $\phi \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. This implies that y $\in \sigma_{\alpha-\beta} - cl(f(A))$. Therefore ,we have $f(\tau_{\alpha-\gamma} - cl(A)) \subseteq \sigma_{\alpha-\beta} - cl(f(A))$.

$(ii) \rightarrow (iii)$. Let V be a α-β-closed set in Y. Then $\sigma_{\alpha-\beta} - cl(V) = V$. By (ii) $f(\tau_{\alpha-\gamma} - cl(f^{-1}(V))) \subseteq \sigma_{\alpha-\beta} - cl(f(f^{-1}(V))) \subseteq \sigma_{\alpha-\beta} - cl(V) = V$ holds. Therefore $\tau_{\alpha-\gamma} - cl(f^{-1}(V)) \subseteq f^{-1}(V)$ and thus $f^{-1}(V) = \tau_{\alpha-\gamma} - cl(f^{-1}(V))$. Hence $f^{-1}(V)$ is α-γ-closed in X.

$(iii) \rightarrow (i)$. Let B be any α-β-open set in Y. Consider V = Y − B. Then V is α-β-closed in Y. By (iii) $f^{-1}(V)$ is α-γ-closed in X. Hence $f^{-1}(B) = X - f^{-1}(Y - B) = X - f^{-1}(V)$ is α-γ-open in X. Hence f is α-(γ,β)- continuous.

Theorem 3.5 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a α-(γ,β)- continuous mapping and injective. If Y is α-β-T_2 (resp.α-β-T_1),then X is α-γ-T_2 (resp.α-γ-T_1).

Proof: Suppose Y is α-β-T_2. Let x and y be two distinct points of X. Then, there exists two α-β-open sets U and V such that f(x)$\in$ U, f(y)$\in$ V and U$\cap$V = ϕ. Since f is α-(γ,β)- continuous, for U and V,there exists two α-γ-open sets W and S such that x $\in$ W and y $\in$ S, f(W)$\subseteq$ U and f(S)$\subseteq$ V,implies that W $\cap$ S = ϕ. Hence X is α-γ-T_2. In a similar way we can prove that X is α-γ-T_1 whenever Y is α-β-T_1.

Theorem 3.6 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \delta)$ be two mappings.

(i) If f is (α-γ, β)- continuous and g is (β, δ)- continuous, then g$\circ$f is (α-γ, δ)- continuous;

(ii) If f is α-(γ, β)- continuous and g is (α-β, δ)- continuous, then g$\circ$f is (α- γ, δ)- continuous;

(iii) If f is α-(γ, β)- continuous and g is α-(β, δ)- continuous, then g $\circ$ f is α- (γ, δ)- continuous;

Proof: Follows from the Definitions 2.20[7] , 4.1[2] and 6.1[2].

4 α-(γ, β)-open mappings

In this section we introduce the concept of α-(γ, β)-open mappings and study some of its basic properties.
Let (X,τ) and (Y,σ) be two topological spaces. $\gamma : \tau \rightarrow P(X)$ and $\beta : \sigma \rightarrow P(Y)$ be operations on τ and σ respectively.

Definition 4.1 A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be α-(γ, β)-open if and only if for each A $\in \tau_{\alpha-\gamma}$, $f(A) \in \sigma_{\alpha-\beta}$.

Example 4.2 Let X = $\{a, b, c\}$, Y = $\{1, 2, 3\}$, $\tau = \{\varphi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{\varphi, Y, \{2\}, \{1, 3\}\}$. Define operations $\gamma : \tau \to P(X)$ and $\beta : \sigma \to P(Y)$ by $A^\gamma = cl(A)$ for every A $\in \tau$ and $B^\beta = cl(B)$ for every B $\in \sigma$.
Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = 1, f(b) = 3$ *and* $f(c) = 2$. The image of every α-γ-open set is α-β-open under f. Hence f is α-(γ , β)- open.

Remark 4.3 Every α-(γ, β)-open mapping is (γ, α-β)-open. But the converse need not be true.

Let X = $\{a, b, c\}$, Y = $\{1, 2, 3\}$, $\tau = \{\varphi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{\varphi, Y, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}\}$.
Define Operations $\gamma : \tau \to P(X)$ and $\beta : Y \to P(Y)$ by

$$A^\gamma = \begin{cases} A & if A = \{a\} \\ A \cup \{c\} & if A \neq \{a\} \end{cases}$$

$$B^\beta = \begin{cases} cl(B) & if b \in B \\ B & if b \notin B \end{cases}$$

Define $f : (X, \tau) \to (Y, \sigma)$ by $f(a) = 1, f(b) = 1$ *and* $f(c) = 2$. The image of every γ-open set in X is α-β-open in Y under f. Hence f is (γ , α-β)- open. But the image of every α-γ-open set is not α-β-open. Hence f is not α-(γ, β) -open.

Remark 4.4 If $f : (X, \tau) \to (Y, \sigma)$ is said to be α-(γ, β)-open and $g : (Y, \sigma) \to (Z, \delta)$ is α-(β, δ)- open, then the composition $g \circ f : (X, \tau) \to (Z, \delta)$ is α-(γ, δ)-open mapping.

Theorem 4.5 A mapping $f : (X, \tau) \to (Y, \sigma)$ is α-(γ, β)-open if and only if for each x $\in$ X, and for every A $\in \tau_{\alpha-\gamma}$ such that x $\in$ A, there exists B $\in \sigma_{\alpha-\beta}$ such that $f(x) \in$ B and B $\subseteq f(A)$.

Proof: Let A be a α-γ-open set of x $\in$ X. Then $f(x) \in f(A)$. Therefore $f(A)$ is a α-β-open neighbourhood of $f(x)$ in Y. Then by Theorem 3.3 there exists a α-γ-open neighbourhood B $\in \sigma_{\alpha-\beta}$ such that $f(x) \in B \subseteq f(A)$.

Conversely, Let A $\in \tau_{\alpha-\gamma}$ such that x $\in$ A. Then by assumption, there exists B $\in \sigma_{\alpha-\beta}$ such that $f(x) \in$ B $\subseteq f(A)$. Therefore $f(A)$ is a α- β-neighbourhood of $f(x)$ in Y and this implies that $f(A) = \cup_{f(x) \in f(A)} B$. Then by Theorem 3.4 [8] $f(A)$ is α- β-open in Y. Hence f is α-(γ, β)-open.

Theorem 4.6 A mapping $f : (X, \tau) \to (Y, \sigma)$ is α-(γ, β)-open if and only if for each x $\in$ X, and for every α-γ-neighbourhood U of x $\in$ X there exists a α- β-neighbourhood V of $f(x) \in$ Y such that $V \subseteq f(U)$.

Proof: Let U be a α-γ-neighbourhood of x $\in$ X. Then by Definition 3.1 there exists a α-γ-open set W such that x $\in$ W $\subseteq$ U. This implies that $f(x) \in f(W) \subseteq f(U)$. Since f is a α-(γ, β)-open mapping ,we have $f(W)$ is α- β-open. Hence V = f(W) is a α- β-neighbourhood of f(x) and V $\subseteq$ f(U).

Conversely, Let U $\in \tau_{\alpha-\gamma}$ and x $\in$ U. Then U is a α-γ-neighbourhood of x . So by hypothesis, there exists a α- β-neighbourhood V of f(x) such that f(x) $\in$ V $\subseteq$ f(U). That is , f(U) is a α- β-neighbourhood of f(x). Thus f(U) is a α- β-heighbourhood of each of its points. Therefore f(U) is α- β-open. Hence f is α-(γ, β)-open.

Theorem 4.7 A mapping $f : (X,\tau) \to (Y,\sigma)$ is α-(γ, β)-open if and only if $f(\tau_{\alpha-\gamma} - int(A)) \subseteq \sigma_{\alpha-\beta} - int(f(A))$, for all A $\subseteq$ X.

Proof: Let x $\in \tau_{\alpha-\gamma} - int(A)$. Then there exists U $\in \tau_{\alpha-\gamma}$ such that x $\in$ U $\subseteq$ A. So f(x) $\in$ f(U) $\subseteq$ f(A). Since f is α-(γ, β)-open, f(U) is α-β-open in Y. Hence f(x) $\in \sigma_{\alpha-\beta} - int(f(A))$. Thus $f(\tau_{\alpha-\gamma} - int(A)) \subseteq \sigma_{\alpha-\beta} - int(f(A))$.

Conversely, Let U $\in \tau_{\alpha-\gamma}$. Then by hypothesis , $f(U) = f(\tau_{\alpha-\gamma} - int(U)) \subseteq \sigma_{\alpha-\beta} - int(f(U)) \subseteq f(U)$ or $f(U) \subseteq \sigma_{\alpha-\beta} - int(f(U)) \subseteq f(U)$. This implies that f(U) is α- β-open. So f is α-(γ, β)-open.

Theorem 4.8 A mapping $f : (X,\tau) \to (Y,\sigma)$ is α-(γ, β)-open if and only if $\tau_{\alpha-\gamma} - int(f^{-1}(B)) \subseteq f^{-1}(\sigma_{\alpha-\beta} - int(B))$, for all B $\subseteq$ Y.

Proof: Let B be any subset of Y. Clearly,$\tau_{\alpha-\gamma} - int(f^{-1}(B))$ is α-γ-open in X. Also $f(\tau_{\alpha-\gamma} - int(f^{-1}(B)) \subseteq f(f^{-1}(B)) \subseteq B$. Since f is α-(γ, β)-open and by Theorem 4.7 ,we have $f(\tau_{\alpha-\gamma} - int(f^{-1}(B)) \subseteq \sigma_{\alpha-\beta} - int(B)$. Hence $\tau_{\alpha-\gamma} - int(f^{-1}(B)) \subseteq f^{-1}(f(\tau_{\alpha-\gamma} - int(f^{-1}(B)))) \subseteq \sigma_{\alpha-\beta} - int(B)$. This implies that $\tau_{\alpha-\gamma} - int(f^{-1}(B)) \subseteq f^{-1}(\sigma_{\alpha-\beta} - int(B))$ for all B $\subseteq$ Y.

Conversely, Let $A \subseteq X$. By hypothesis, we obtain $\tau_{\alpha-\gamma} - int(A) \subseteq \tau_{\alpha-\gamma} - int(f^{-1}(f(A))) \subseteq f^{-1}(\sigma_{\alpha-\beta} - int(f(A)))$. This implies that $f(\tau_{\alpha-\gamma} - int(A)) \subseteq f(\tau_{\alpha-\gamma} - int(f^{-1}(f(A)))) \subseteq f(f^{-1}(\sigma_{\alpha-\beta} - int((f(A)))) \subseteq \sigma_{\alpha-\beta} - int((f(A))$. Consequently, $f(\tau_{\alpha-\gamma} - int(A)) \subseteq \sigma_{\alpha-\beta} - int((f(A))$, for all A $\subseteq$ X. By Theorem 4.7, f is α-(γ, β)-open .

Theorem 4.9 A mapping $f : (X,\tau) \to (Y,\sigma)$ is α-(γ, β)-open if and only if $f^{-1}(\sigma_{\alpha-\beta} - cl(B)) \subseteq \tau_{\alpha-\gamma} - cl(f^{-1}(B))$, for all B $\subseteq$ Y.

Proof: Let B be any subset of Y. By theorem 4.8 $\tau_{\alpha-\gamma} - int(f^{-1}(Y - B)) \subseteq f^{-1}(\sigma_{\alpha-\beta} - int(Y - B))$. Then $\tau_{\alpha-\gamma} - int(X - f^{-1}(B)) \subseteq f^{-1}(\sigma_{\alpha-\beta} - int(Y - B))$. As $\sigma_{\alpha-\beta} - int(B)) = Y - \sigma_{\alpha-\beta} - cl(Y - B))$, therefore $X - \tau_{\alpha-\gamma} - cl(f^{-1}(B) \subseteq f^{-1}(Y - \sigma_{\alpha-\beta} - cl((B))$ or $X - \tau_{\alpha-\gamma} - cl(f^{-1}(B) \subseteq X - f^{-1}(\sigma_{\alpha-\beta} - cl(B))$. Hence $f^{-1}(Y - \sigma_{\alpha-\beta} - cl(B)) \subseteq \tau_{\alpha-\gamma} - cl(f^{-1}(B)$.

Conversely, Let B $\subseteq$ Y. By hypothesis, $f^{-1}(\sigma_{\alpha-\beta} - cl(Y - B)) \subseteq \tau_{\alpha-\gamma} - cl(f^{-1}(Y - B))$. Then

$X-\tau_{\alpha-\gamma}-cl(f^{-1}(Y-B)) \subseteq X-f^{-1}(\sigma_{\alpha-\beta}-cl(Y-B))$. Hence $X-\tau_{\alpha-\gamma}-cl(X-f^{-1}(B)) \subseteq f^{-1}(Y-\sigma_{\alpha-\beta}-cl(Y-B))$. This gives that $\tau_{\alpha-\gamma}-int(f^{-1}(B)) \subseteq f^{-1}(\sigma_{\alpha-\beta}-int(B))$. Using Theorem 4.8, it follows that f is α-(γ, β)-open .

Theorem 4.10 Let $f : (X,\tau) \rightarrow (Y,\sigma)$ and $g : (Y,\sigma) \rightarrow (Z,\zeta)$ be two mappings such that $g \circ f : (X,\tau) \rightarrow (Z,\delta)$ be α-(γ, β)- continuous mapping. Then
(i) If g is α-(β, δ)-open injection then f is α-(β, δ)- continuous;

(ii) If f is α-(γ ,β)-open surjection then g is α-(β, δ)- continuous;

Proof: (i) Let U $\in \sigma_{\alpha-\beta}$. Since g is α-(β, δ)-open, then g(U) $\in \zeta_{\alpha-\delta}$. Since g is injective and $g \circ f$ is α-(γ, δ)-continuous ,we have $(g \circ f)^{-1}$ (g(U))= $(f^{-1} \circ g^{-1})$ (g(U)) $= f^{-1}(g^{-1}g(U)) = f^{-1}(U)$ is α-γ-open in X. this proves that f is α-(γ, β)-continuous.

(ii) Let V $\in \zeta_{\alpha-\delta}$. Since $g \circ f$ is α-(γ, δ)-continuous, then $(g \circ f)^{-1}(V) \in \tau_{\alpha-\gamma}(X)$. Also f is α-(γ ,β)-open , so $f((g \circ f)^{-1}(V))$ is α- β-open in Y. Since f is surjective, we obtain $(f \circ (g \circ f)^{-1})(V) = (f \circ (f^{-1} \circ g^{-1}))(V) = ((f \circ f^{-1}) \circ g^{-1})(V) = g^{-1}(V)$. It follows that $g^{-1}(V) \in \sigma_{\alpha-\beta}$. This proves that g is α-(β, δ)- continuous mapping.

5 α-(γ, β)-closed mappings

In this section we introduce the concept of α-(γ, β)-closed mappings and study some of its basic properties.
Let (X,τ) and (Y,σ) be two topological spaces. $\gamma : \tau \rightarrow P(X)$ and $\beta : \sigma \rightarrow P(Y)$ be operations on τ and σ respectively.

Definition 5.1 A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be α-(γ, β)-closed if and only if the image set f(A) is α- β-closed for each α-γ-closed subset A of X.

Example 5.2 Let X $= \{a,b,c\}$, Y $= \{1,2,4\}$, $\tau = \{\varphi, X, \{a\}, \{c\}, \{a,b\}, \{a,c\}\}$ and $\sigma = \{\varphi, Y, \{1\}, \{4\}, \{1,2\}, \{1,4\}\}$ .Define Operations $\gamma : \tau \rightarrow P(X)$ and $\beta : Y \rightarrow P(Y)$ by

$$A^{\gamma} = \begin{cases} A & if A = \{a\} \\ A \cup \{c\} & if A \neq \{a\} \end{cases}$$

$$B^{\beta} = \begin{cases} cl(B) & if b \in B \\ B & if b \notin B \end{cases}$$

Define $f : (X,\tau) \rightarrow (Y,\sigma)$ by $f(a) = 1, f(b) = 2 \quad and \quad f(c) = 2$. The image of every α-γ-closed set in X is α-β-closed in Y under f. Hence f is α-(γ , β)- closed.

Remark 5.3 Every α-(γ, β)-closed mapping is (γ, α-β)-closed. But the converse need not be true.

Let X = $\{a, b, c\}$, Y = $\{1, 2, 3\}$, $\tau = \{\varphi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ and $\sigma = \{\varphi, Y, \{1\}, \{3\}, \{1, 2\}, \{1, 3\}\}$ .Define Operations $\gamma : \tau \rightarrow P(X)$ and $\beta : Y \rightarrow P(Y)$ by

$$A^{\gamma} = \begin{cases} A & if b \notin A \\ cl(A) & if\ b \in A \end{cases}$$

$$B^{\beta} = \begin{cases} cl(B) & if b \notin B \\ B \cup \{c\} & if b \in B \end{cases}$$

Define $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a)=1, f(b) = 3 and f(c) = 2. f is (γ, α-β)-closed but not α-(γ, β)-closed.

Remark 5.4 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is α-(γ , β)- closed and $g : (Y, \sigma) \rightarrow (Z, \zeta)$ is α-(β , δ)- closed , then $g \circ f : (X, \tau) \rightarrow (Z, \delta)$ be α-(γ, δ)- closed.

Definition 5.5 A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be α-(γ , β)-homeomorphism,if f is bijective , α-(γ , β)-continuous and f^{-1} is α-(γ , β)-homeomorphism.

Remark 5.6 From the definitions 6.1[3] and 5.1 every bijective , α-(γ,β)-continuous and α-(γ,β)-closed map is α-(γ,β)-homeomorphism.

Theorem 5.7 A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is α-(γ,β)-closed if and only if $\sigma_{\alpha-\beta} - cl(f(A)) \subseteq f(\tau_{\alpha-\gamma} - cl(A))$, for every subset A of X.

Proof: Suppose f is α-(γ,β)-closed and let $A \subseteq X$. Then $f(\tau_{\alpha-\gamma} - cl(A))$ is α-β-closed in Y. Since f(A) $\subseteq f(\tau_{\alpha-\gamma} - cl(A))$, we obtain $\sigma_{\alpha-\beta} - cl(f(A)) \subseteq f(\tau_{\alpha-\gamma} - cl(A))$.

Conversely, suppose A is a α-γ-closed set in X. By hypothesis ,we obtain $f(A) \subseteq \sigma_{\alpha-\beta} - cl(f(A)) \subseteq f(\tau_{\alpha-\gamma} - cl(A)) = f(A)$. Hence $f(A) = \sigma_{\alpha-\beta} - cl(f(A))$. Thus f(A) is α-β-closed set in Y. This proves that f is α-(γ,β)-closed .

Theorem 5.8 A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is α-(γ,β)-closed if and only if $\sigma_{\beta} - cl(\sigma_{\beta} - int(\sigma_{\beta} - cl(f(A)))) \subseteq f(\tau_{\alpha-\gamma} - cl(A))$, for every subset A of X.

Proof: Suppose f is α-(γ,β)-closed and let A $\subseteq$ X. Then $f(\tau_{\alpha-\gamma} - cl(A))$ is α-β-closed in Y. This implies that $\sigma_{\beta} - cl(\sigma_{\beta} - int(\sigma_{\beta} - cl(f(\tau_{\alpha-\gamma} - cl(A))))) \subseteq f(\tau_{\alpha-\gamma} - cl(A))$. Then $\sigma_{\beta} - cl(\sigma_{\beta} - int(\sigma_{\beta} - cl(f(A)))) \subseteq \sigma_{\beta} - cl(\sigma_{\beta} - int(\sigma_{\beta} - cl(f(\tau_{\alpha-\gamma} - cl(A)))))$ gives $\sigma_{\beta} - cl(\sigma_{\beta} - int(\sigma_{\beta} - cl(f(A)))) \subseteq f(\tau_{\alpha-\gamma} - cl(A))$.

Conversely, Suppose A is a α-γ-closed set in X. Then by hypothesis,$\sigma_{\beta} - cl(\sigma_{\beta} - int(\sigma_{\beta} - cl(f(A)))) \subseteq f(\tau_{\alpha-\gamma} - cl(A))$. Since A is α-γ-closed ,we obtain $f(\tau_{\alpha-\gamma} - cl(A)) \subseteq f(A)$. Hence $f(A)$ is α-β-closed in Y. This implies that f is α-(γ,β)-closed.

Theorem 5.9 A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is α-(γ,β)-closed if and only if for each subset B of Y and each α-γ-open set A in X containing $f^{-1}(B)$, there exists a α-β-open set C in Y containing B such that $f^{-1}(C) \subseteq A$.

proof: Let C = Y $-$ f(X $-$ A). Then $f(X-A) \subseteq Y-B$. Since f is α-(γ,β)-closed , then C is α-β-open and $f^{-1}(C) = X - f^{-1}(f(X-A)) \subseteq X-(X-A) = A$.
Conversely, suppose F is a α-γ-closed set in X. Let B = Y $-$ f(F). Then $f^{-1}(B) \in X - f^{-1}(f(F)) \subseteq X-F$ and X $-$ F is α-γ-open in X. Hence by hypothesis, there exists a α-β-open set C containing y such that $f^{-1}(C) \subseteq X-F$. Then we have $f^{-1}(C) \cap F = \phi$ and $C \cap f(F) = \phi$. Therefore $Y - f(F) \supseteq C \supseteq B = Y - f(F)$ and f(F) is α-β- closed in Y. This proves that f is α-(γ,β)-closed.

Theorem 5.10 Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be a bijective mapping. Then the following are equaivalent:

(i) f is α-(γ,β)-closed.

(ii) f is α-(γ,β)-open.

(iii) f^{-1} is α-(γ,β)-continuous.

Proof $(i) \Rightarrow (ii)$ Follows from the Definitions 7.1 and 8.1.

$(ii) \Rightarrow (iii)$ Let A be a α-γ-closed set in X. Then $\tau_{\alpha-\gamma} - cl(A) \subseteq A$. By Condition (ii) and by Theorem 4.9 , $f^{-1}(\sigma_{\alpha-\beta} - cl(f(A))) \subseteq \tau_{\alpha-\gamma} - cl(f^{-1}(f(A)))$ implies that $\sigma_{\alpha-\beta} - cl(f(A)) \subseteq f(\tau_{\alpha-\gamma} - cl(A))$. Thus $\sigma_{\alpha-\beta} - cl((f^{-1})^{-1}(A)) \subseteq (f^{-1})^{-1}(A)$, for every subset A of X, it follows that f^{-1} is α-(γ,β)-continuous.

$(iii) \Rightarrow (i)$. Let A be a α-γ-closed set in X. Then X $-$ A is α-γ-open in X. Since f^{-1} is α-(γ,β)-continuous, $(f^{-1})^{-1}(X-A))$ is α-β-open set in Y. But $(f^{-1})^{-1}(X-A)) = f(X-A) = Y - f(A)$. Thus f(A) is α-β-closed in Y. This proves that f is α-(γ,β)-closed.

Definition 5.11 Let id: $\tau \rightarrow P(X)$ be the identity operation . A mapping $f : (X,\tau) \rightarrow (Y,\sigma)$ is said to be α-(id,β)-closed if for any α-closed set F of X, f(F) is α-β-closed in Y.

Definition 5.12 If f is bijective mapping and $f^{-1} : (Y,\sigma) \rightarrow (X,\tau)$ is α-(id,β)-continuous ,then f is α-(id,β)-closed .

Proof: Follows from the Definitions 6.1[2], 5.1 and 5.5.

Theorem 5.13 Suppose that f is α-(γ,β)-continuous mapping and A is α-(γ,β)-closed. Then

(i) For every α-γ g-closed set A of (X,τ) the image f (A) is α-β g-closed.

(ii) For every α-β g-closed set B of (Y,σ) , the set $f^{-1}(B)$ is α-γ g-closed.

Proof: (i) Let V be any α-β-open set in Y such that $f(A) \subseteq V$. By using Theorem 3.3 $f^{-1}(V)$ is a α-γ-open set containing A. Therefore by assumption we have $\tau_{\alpha-\gamma} - cl(A) \subseteq f^{-1}(V)$, so $f(\tau_{\alpha-\gamma} - cl(A))) \subseteq V$. Since f is α-(γ,β)-closed, $f(\tau_{\alpha-\gamma} - cl(A)))$ is a α-β-closed set containing f (A), implies that $\sigma_{\alpha-\beta} - cl(f(A)) \subseteq \sigma_{\alpha-\beta} - cl(f(\tau_{\alpha-\gamma} - cl(A)))) = f(\tau_{\alpha-\gamma} - cl(A))) \subseteq V$. Hence f (A) is α-β g-closed.

(ii) Let U be a α-γ-open set of (X,τ) such that $f^{-1}(B) \subseteq U$ for any subset B in Y. Put $F = \tau_{\alpha-\gamma} - cl(f^{-1}(B)) \cap (X - U)$. It follows from remark 3.23 (ii) [8] and Theorem 3.4 [9] that F is α-γ-closed set A in (X,τ) . Since f is α-(γ,β)-closed , f (F) is α-β-closed in (Y,σ) . By using Theorem 4.8 [9], Theorem 3.4 (ii) and the following inclusion $f(F) \subseteq \sigma_{\alpha-\beta} - cl(B) - B$, it is obtained that f (F) = ϕ,and hence F = ϕ . This implies that $\tau_{\alpha-\gamma} - cl(f^{-1}(B)) \subseteq U$. Therefore $f^{-1}(B)$ is α-γ g - closed.

Theorem 5.14 Let $f : (X, \tau) \to (Y, \sigma)$ is α-(γ,β)-continuous and α-(γ,β) closed. Then

(i) If f is injective and (Y,σ) is α-β-T$_{\frac{1}{2}}$ then (X,τ) is α-γ-T$_{\frac{1}{2}}$ space.

(ii) If f is surjective and (X,τ) is α-γ-T$_{\frac{1}{2}}$ then (Y,σ) is α-β-T$_{\frac{1}{2}}$ space.

Proof: (i) Let A is α-γ g - closed set in (X,τ) .Then by Theorem 5.13 (i) f (A) is α-β g -closed. Therefore by assumption A is α-γ-closed in (X,τ) . Therefore (X,τ) is α-γ-T$_{\frac{1}{2}}$ space.

(ii) Let B be α-β g -closed set in (Y,σ) .Then it follows from Theorem 5.13 (ii) and the assumption that $f^{-1}(B)$ is α-γ-closed. Hence f is α-(γ,β)-closed map, implies that $f(f^{-1}(B)) = B$ is α-β-closed in (Y,σ). Therefore (Y,σ) is α-β-T$_{\frac{1}{2}}$.

Theorem 5.15 Let $f : (X, \tau) \to (Y, \sigma)$ is α-(γ,β)-homeomorphism. If (X,τ) is α-γ-T$_{\frac{1}{2}}$ then (Y,σ) is α-β-T$_{\frac{1}{2}}$.

Proof: Let $\{y\}$ be a singleton set of (Y,σ) . Then there exists a point x of X such that y = f (x). By Theorem 4.10[9],it follows that the singleton set $\{y\}$ is α-β-open or α-β-closed. Therefore (Y,σ) is α-β-T$_{\frac{1}{2}}$ space.

Theorem 5.16 Let $f : (X, \tau) \to (Y, \sigma)$ is α-(γ,β)-continuous, injective mapping . If (Y, σ) is α-β-T_1 space (respectively α-β-T_2) then (X, τ) is α-γ-T_1 space (respectively α-γ-T_2) .

Proof: Suppose (Y,σ) is α-β- T_2 space and x,y be two distinct points in X . Then there exists two α-β-open sets V and W of Y such that f(x)$\in$ V and f(y) $\in$ W and $V \cap W = \phi$. Since, f is α-(γ,β)-continuous for V and W there exists two α-γ-open sets U and S such that x $\in$ U and y $\in$ S and f(U) $\subseteq$ V and f(S) $\subseteq$ W. Therefore $U \cap S = \phi$. Hence (X,τ) is α-γ-T_2 space. The proof of the case α-γ-T_1 is proved similarly.

Definition 5.17 If $\gamma : \tau \to P(X)$ is a regular operation then X is a α-γ-$T_{\frac{1}{2}}$ space.

Proof: By proposition 2.9 [7] , we have (X, τ_γ) is a topological space. To prove X is α-γ-$T_{\frac{1}{2}}$ space, it is enough to show that $\{x\}$ is α-γ-open or α-γ-closed.

Case (i): Suppose $\{x\} \in \tau_\gamma$, then by Theorem 3.17[9] $\{x\}$ is α-γ-open.

Case (ii): Suppose $\{x\} \notin \tau_\gamma$, then $\tau_\gamma - int(\tau_\gamma - cl(\tau_\gamma - int(\{x\}))) = \tau_\gamma - cl(\phi) = \phi \subseteq \{x\}$. Hence $\{x\}$ is α-γ-closed.

Definition 5.18 Let (X,τ) be a topological space and γ be an operation on τ . Then a subset A of X is said to be α-γ generalized open set (α-γ g-open set) if $F \subseteq \tau_{\alpha-\gamma} - int(A)$ whenever F $\subseteq$ A and F is α-γ- closed in (X,τ) . A subset A of X is said to be α-γ g-closed if X $-$ A is α-γ g-open .

The family of all α-γ generalized open set in (X,τ) is denoted by $\tau_{\alpha-\gamma g}$-open set and the family of all α-γ generalized closed set in (X,τ) is denoted by $\tau_{\alpha-\gamma g}$-closed set.

Remark 5.19 The union of two disjoint α-γ g-closed set need not be a α-γ g- closed set.

Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$,define an operation γ on τ such that

$$A^\gamma = \begin{cases} cl(A) & if \;\; b \in A \\ A & if \;\; b \notin A \end{cases}$$

then $\tau_{\alpha-\gamma} = \{\phi, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$, α-γ g-closed set $= \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. $A = \{a\}$ and $B = \{b\}$ are α-γ g-closed sets but $A \cup B = \{a, b\}$ is not a α-γ g-closed set.

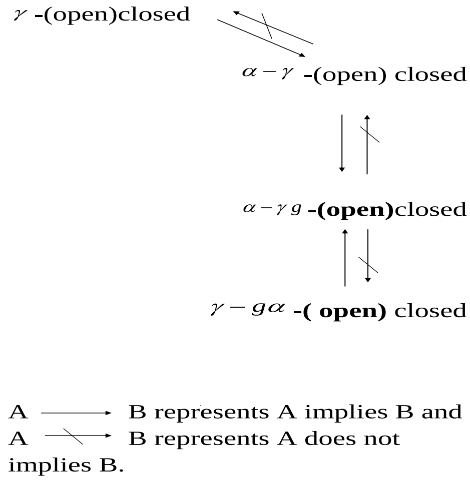
6 γ-g α-open sets

Definition 6.1 Let (X,τ) be a topological space and γ be an operation on τ . Then a subset A of X is said to be γ generalized α- open set (γ-g α-open set) if F $\subseteq \tau_{\alpha-\gamma} - int(A)$ whenever F $\subseteq$ A and F is γ-closed in (X,τ) . A subset A of X is said to be γ-g α-closed if X $-$ A is γ-g α-open.

Theorem 6.2 Let (X,τ) be a topological space and γ be an operation on τ . Then a subset A of X is said to be γ-g α-closed set if and only if $\tau_{\alpha-\gamma} - cl(A) \subseteq U$ whenever A $\subseteq$ U and U is γ- open in (X,τ) .

Proof: Proof follows from the Definition 6.1 and the results $\tau_{\alpha-\gamma} - int(A) = X - \tau_{\alpha-\gamma} - cl(A)$, $\tau_{\alpha-\gamma} - cl(A) = X - \tau_{\alpha-\gamma} - int(A)$

Remark 6.3 From the Definitions 4.6 [9], 5.18 and 6.1 have the following digrammatic implications:



Remark 6.4 The union of two disjoint γ-g α-closed sets need not be a γ-g α-closed set.

Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$,define an operation γ on τ such that

$$A^{\gamma} = \begin{cases} cl(A) & if \;\; b \notin A \\ A & if \;\; b \in A \end{cases}$$

Then $\tau_{\alpha-\gamma} = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. α-γ g-closed set $= \{\phi, X, \{b\}, \{c\}, \{a, c\}\}$. $A = \{b\}$ and $B = \{c\}$ are α-γ g-closed sets .But $A \cup B = \{b, c\}$ is not a α-γ g-closed set.

Theorem 6.5 Let (X,τ) be a topological space and γ be an operation on τ . If A is γ-open and γ-g α-closed set in (X,τ), then A is α-γ-closed.

Proof: Since A is γ-open and γ-g α-closed , $\tau_{\alpha-\gamma} - cl(A) \subseteq A$ and hence $\tau_{\alpha-\gamma} - cl(A) = A$. This implies that A is α-γ-closed.

Theorem 6.6 Let (X,τ) be a topological space and γ be an operation on τ. If A is γ-g α-closed set in (X,τ) , then $\tau_{\alpha-\gamma} - cl(A)$ − A does not contain any nonempty γ-closed set.

Proof: Let F be a γ-closed sub set of $\tau_{\alpha-\gamma} - cl(A) - A$. This implies that $A \subseteq (X - F)$. Since A is γ-g α-closed and X − F is γ-open, implies $\tau_{\alpha-\gamma} - cl(A) \subseteq$(X - F) . Therefore we have $F \subseteq (X - \tau_{\alpha-\gamma} - cl(A)) \cap (\tau_{\alpha-\gamma} - cl(A)) = \phi$. Hence $F = \phi$.

Theorem 6.7 Let (X,τ)be a topological space and γ be an operation on τ. Then for each x $\in$ X, $\{x\}$ is γ-closed or X − $\{x\}$ is γ-g α-closed in (X,τ) .

Proof: Suppose $\{x\}$ is not γ-closed. Then X $-$ $\{x\}$ is not a γ-open set. Therefore X is the only γ- open set containing X $-$ $\{x\}$. Hence we have $\tau_{\alpha-\gamma} - cl(X - \{x\}) \subseteq X$. This implies X $-$ $\{x\}$ is γ-g α-closed .

Theorem 6.8 Let (X,τ) be a topological space and γ be a regular operation on τ . Then the following are equivalent :
(i) Every γ-g α-closed set of (X,τ) is α-γ-closed.
(ii) For each x $\in$ X , $\{x\}$ is γ-closed or α-γ-open in (X,τ) .
(iii) (X,τ) is α-γ-$T_{\frac{1}{2}}$-space.

Proof: $(i) \rightarrow (ii)$ Suppose that for an x $\in$ X ,$\{x\}$ is not γ-closed. By Theorem 6.7, X $-$ $\{x\}$ is a γ-g α- closed set. Therefore by assumption X $-$ $\{x\}$ is α-γ-closed. Hence $\{x\}$ is α-γ-open.

$(ii) \rightarrow (iii)$ By Theorem 6.7 X $-$ $\{x\}$ is γ-g α-closed, using Theorem 4.6 [8] and 4.10[8] ,(X,τ) is α-γ-$T_{\frac{1}{2}}$-space.

$(iii) \rightarrow (i)$ By Theorem 6.5.

Definition 6.9 A topological space (X,τ) is said to be γ-α T_b space (respectively γ-α T_d space) if every γ-g α-closed set is γ-closed (respectively γ- g.closed).

Theorem 6.10 (i) If (X,τ) is γ-α T_b ,then for each x $\in$ X ,$\{x\}$ is α-γ-closed or γ- open.

(ii) If (X,τ) is γ-α T_d , then for each x $\in$ X, $\{x\}$ is γ-closed or γ-g.open.

Proof: (i) Suppose that for x $\in$ X , $\{x\}$ is not α-γ-closed , then by Theorem 6.7, X $-$ $\{x\}$ is γ-g α-closed. Therefore , by assumption X $-$ $\{x\}$ is γ-closed. Hence $\{x\}$ is γ-open.

(ii) Suppose that , for x $\in$ X , $\{x\}$ is not γ-closed. Then by Theorem 6.7 and by the assumption it follows that X $-$ $\{x\}$ is γ-g α-closed and X $-$ $\{x\}$ is γ-g.closed. Hence $\{x\}$ is γ-g.open.

Remark 6.11 Let (X,τ) be a topological space and γ be a regular operation on τ . Then every γ-α T_b space is γ-α T_d and α-γ-$T_{\frac{1}{2}}$-space. However the converse need not be true.

Proof: Proof follows from the Definition 6.9 and Theorem 6.10 .

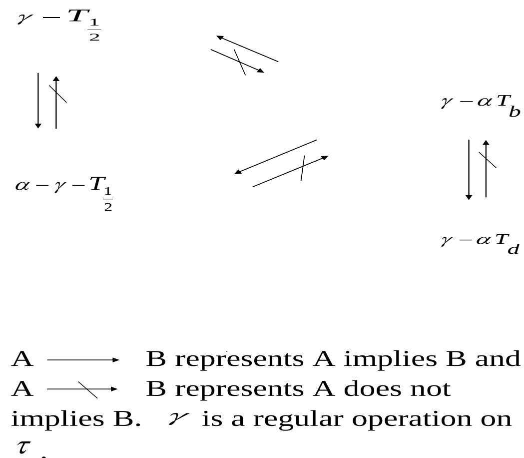
Let $X = \{a, b, c\}$,$\tau = \{\phi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$,define an operation γ on τ such that

$$A^{\gamma} = \begin{cases} cl(A) & if \ \ b \notin A \\ A & if \ \ b \in A \end{cases}$$

Then $\tau_{\gamma} = \{\phi, X, \{b\}, \{a, c\}, \{b, c\}\}$, $\tau_{\alpha-\gamma} = \{\phi, X, \{b\}, \{a, c\}, \{b, c\}\}$. (X,$\tau$) is a α-γ-$T_{\frac{1}{2}}$-

space but not a γ-α T_b and γ-α T_d space.

Remark 6.12 From the Definition 4.6[8], Definition 6.9 and the Remark 6.11, we have the following diagram implications:



Definition 6.13 A topological space (X,τ) is called γ- $T_{g\alpha}$ space if for every γ-g α-closed set is α-γ g-closed.

Remark 6.14 Let (X,τ) be a topological space and be γ a regular operation on τ . Then by Definitions 5.18, 6.1 and 6.13 ,every α-γ-$T_{\frac{1}{2}}$-space is γ- $T_{g\alpha}$ space .

Theorem 6.15 If $f : (X, \tau) \rightarrow (Y, \sigma)$ is (γ,β) continuous and α-(γ,β)-closed, then for every γ-g α-closed set B of (X,τ) , f (B) is β-g α-closed in (Y,σ).

bf Proof: Let U be a β-open set such that f (B)$\subseteq$ U. Then $B \subseteq f^{-1}(U)$. Since f is (γ,β)-continuous and B is γ-g α-closed set, implies $\tau_{\alpha-\gamma} - cl(B) \subseteq f^{-1}(U)$ and hence $f(\tau_{\alpha-\gamma} - cl(B)) \subseteq U$. Therefore it follows from the assumption that $\tau_{\alpha-\gamma} - cl(f(B)) \subseteq f(\tau_{\alpha-\gamma} - cl(B)) \subseteq U$. Hence f (B) is β-g α-closed in (Y, σ) .

Theorem 6.16 Let $f : (X, \tau) \rightarrow (Y, \sigma)$ is α-(γ,β)-continuous and α-(γ,β)-closed, then for every β-g α-closed set A of (Y, σ), $f^{-1}(A)$ is γ-g α-closed in (X,τ) .

Proof: Let A be a β-g α-closed set in (Y, σ) . Let U be a γ-open set such that $f^{-1}(A) \subseteq U$. Since f is α-(γ,β)-continuous , $f(\tau_{\alpha-\gamma} - cl(f^{-1}(A))) \bigcap (X - U) \subseteq f(\tau_{\alpha-\gamma} - cl(f^{-1}(A))) \bigcap f(X - U) \subseteq \sigma_{\alpha-\beta} - cl(f^{-1}(A)) \cap (X - A) \subseteq \sigma_{\alpha-\beta} - cl(A) - A$. Since f is α-(γ-β)-closed and $\sigma_{\alpha-\beta} -$

$cl(f^{-1}(A)) \cap (X-U)$ is a α- γ-closed set, implies $\sigma_{\alpha-\beta} - cl(A) - A$ contains a α-β-closed set $f(\tau_{\alpha-\gamma} - cl(f^{-1}(A))) \bigcap (X-U)$. Hence by Theorem 4.7 [8] $f(\tau_{\alpha-\gamma} - cl(f^{-1}(A))) \bigcap (X-U) = \phi$. This implies that $f(\tau_{\alpha-\gamma} - cl(f^{-1}(A))) \bigcap (X-U) = \phi$, hence $\tau_{\alpha-\gamma} - cl(f^{-1}(A)) \subseteq U$. Therefore, $f^{-1}(A)$ is γ-g α-closed.

Theorem 6.17 Let $f : (X, \tau) \to (Y, \sigma)$ is a α-(γ,β)-homeomorphism. If (X,τ) is a γ-$T_{g\alpha}$ space then (Y,σ) is β-$T_{g\alpha}$ space.

Proof: Let F be a β-g α-closed set in (Y,σ) . Then by assumption and Theorem 6.16 we have $f^{-1}(F)$ is γ-g α-closed. Then by Theorem 6.15 $f(f^{-1}(F)) = F$ is β-g α-closed. Therefore (Y,σ) is β-$T_{g\alpha}$ space.

Theorem 6.18 If (X,τ) is γ-α T_b, $f : (X, \tau) \to (Y, \sigma)$ is a α-(γ,β)-homeomorphism and (γ,β)-closed map, then (Y,σ) is β-α T_b.

Proof: Let F be a γ-g α-closed set in (Y,σ) then by Theorem 6.16 $f^{-1}(F)$ is γ-g α-closed in (X,τ) . Since (X,τ) is γ-α T_b and f is (γ,β)-closed , implies that $f^{-1}(F)$ is γ-closed and hence $f^{-1}(F) = F$ is β-closed in (Y,σ). Therefore (Y,σ) is β-α T_b .

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